

UBC Number Theory Seminar: December 8, 2021

Speaker: Prajeet Bajpai (UBC)

Title: Effective Methods for Norm-Form Equations

Abstract: Let $\alpha_1, \dots, \alpha_k$ be linearly independent elements of a number field K of degree $n \geq k$, and let m be an integer. The equation $\text{Norm}_{K/\mathbb{Q}}(x_1\alpha_1 + \dots + x_k\alpha_k) = m$, to be solved in integers, is called a ‘norm-form equation’. The case of binary forms was solved by Thue in 1909, and the general case was resolved by Schmidt in 1971 through his Subspace Theorem generalizing the work of Thue-Siegel-Roth. Unfortunately these results are ineffective, and do not provide any means of determining a bound on the height of exceptional solutions— in particular, they do not allow us to determine a complete list of solutions for even a single norm-form equation.

Baker’s theorem on linear forms in logarithms gave an effective version of Thue’s result for binary forms, and Vojta in his PhD thesis was able extend this effectivity to three-variable norm-form equations under the assumption that K is totally complex and Galois. In this talk we discuss effective resolution for certain norm-form equations in four and five variables, extending the work of Vojta. In particular, we completely and effectively resolve the question of norm-form equations over totally complex Galois sextic fields. The results are motivated by joint work with Mike Bennett.