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Speaker: Lea Beneish (University of California, Berkeley)
Title: Fields generated by points on superelliptic curves
Abstract: We give an asymptotic lower bound on the number of field extensions generated by algebraic points on superelliptic curves over $\mathbb{Q}$ with fixed degree n, discriminant bounded by X, and Galois closure S_n. For C a fixed curve given by an affine equation $y^m = f(x)$ where $m \geq 2$ and $deg f(x) = d \geq m$, we find that for all degrees n divisible by $gcd(m, d)$ and sufficiently large, the number of such fields is asymptotically bounded below by X^{c_n} , where $c_n \to 1/m^2$ as $n \to \infty$. This bound is determined explicitly by parameterizing x and y by rational functions, counting specializations, and accounting for multiplicity. We then give geometric heuristics suggesting that for n not divisible by $\gcd(m, d)$, degree n points may be less abundant than those for which n is divisible by $\gcd(m, d)$. Namely, we discuss the obvious geometric sources from which we expect to find points on C and discuss the relationship between these sources and our parametrization. When one a priori has a point on C of degree not divisible by $\gcd(m, d)$, we argue that a similar counting argument applies. As a proof of concept we show in the case that C has a rational point that our methods can be extended to bound the number of fields generated by a degree n point of C, regardless of divisibility of n by $\gcd(m, d)$. This talk is based on joint work with Christopher Keyes.