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UBC Number Theory Seminar: February 2, 2022

Speaker: Kelly Isham (Colgate University)

Title: Zeta functions and asymptotics related to subrings in $\mathbb{Z}^n$

Abstract: We can define a zeta function of a group (or ring) to be the Dirichlet series associated to the sequence that counts the number of subgroups (or subrings) of a given index. The subgroup zeta function over $\mathbb{Z}^n$ is well-understood, as is the asymptotic growth of subgroups in $\mathbb{Z}^n$. Much less is known about the subring zeta function over $\mathbb{Z}^n$ and the asymptotic growth of subrings in $\mathbb{Z}^n$. In this talk, we discuss the progress toward answering this question and we give new lower bounds on the asymptotic growth of subrings in $\mathbb{Z}^n$. We also define a similar zeta function corresponding to subrings of corank at most k in $\mathbb{Z}^n$. While the proportion of subgroups in $\mathbb{Z}^n$ of corank k is positive for each k, we show this is not the case for subrings in $\mathbb{Z}^n$ of corank k when n is sufficiently larger than k. Lastly, we make connections to orders in number fields. Part of this work is joint with Nathan Kaplan.